A Theoretical Analysis of Public Funding for Research

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Abstract

This paper studies government funding for scientific research. Funds must be distributed among different research institutions and allocated between basic and applied research. Informational constraints prevent less productive institutions to be given any government funding. In order to internalise the beneficial effects of research, the government requires the most productive institutions to carry out more applied research than they would like. Funding for basic research is used by the government to induce more productive institutions to carry out more applied research then they would like.

1 Introduction

A very large amount of taxpayers' money is spent on scientific research. In 2008 in the OECD countries, government expenditure on R&D amounted to around 0.8% of GDP (OECD 2009). These funds are channelled in many different ways, from dedicated research centres, to universities and similar public or private education providers, to subsidies to private non-profit or profit making organisations. Also varied is the link between the funds provided and their destination: some funding is linked to specific research projects, some is simply awarded to institutions to spend as they see fit, some is distributed in consideration of past achievement.

This variety raises immediate efficiency questions. How should the total funding be allocated across different institutions? Is the multiplicity of manners in which these sums of money are assigned a good thing? Would it be possible to re-allocate funding from one spending method to another and improve its impact on society? Should the funding agency be concerned with the nature, basic or applied, of the research carried out by the institutions which it funds? This paper provides a theoretical framework to address these questions: the aim here is to provide a theory of the optimal public research spending.

The approach is microeconomic: I leave the macroeconomic aspect of total spending in the background, and concentrate instead on the two interrelated questions of the *balance between basic and applied* research and of the *distribution of funding among different research institutions*. I make the plausible assumption that research institutions differ in their characteristics. Differences among institutions create a non-trivial optimisation problem: the government wishes to allocate resources to the institutions where they are most productive, and at the same time to ensure the "right" balance between basic and applied research. As my analysis shows, these two requirements interact with each other: the government uses *basic* research funding – more precisely, funding that the recipient institutions will choose to devote to basic research –, as a reward to induce more productive institutions to do more *applied* research. If, plausibly, the government's and the institutions' objectives are not perfectly aligned, this generates a distortion from an efficient allocation, that is an allocation where the social marginal benefit is the same for basic and applied

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research and the same for all institutions. A further distortion is the concentration of research towards the most effective institutions. Relatively inefficient institutions do not receive any funding even though small scale projects would be cheaper to carry out than in larger funded institutions.

The relative role of applied and basic research, at the centre of my study, requires their specific characteristics to be carefully identified and modelled accurately. The distinguishing feature I posit in this paper is that *it is harder to observe whether basic research has been been carried out than it is for applied research.* This relates closely to the definitions offered in the literature. Typically, basic research, also labelled fundamental, pure, curiosity-driven, upstream, unpredictable (see Strandburg 2005), is seen as driven by scientists' curiosity, its aim to acquire knowledge for knowledge's sake, in contrast to applied research, designed to solve practical problems.[1] In many cases a hierarchical link is posited between basic and applied research: the former precedes and provides the foundation to the latter (for example, Evenson and Kislev 1976, or more recently Aghion et al. 2008). Since my model is static, the hierarchical link I posit is not temporal, but in the nature of the connection between research and its effect: applied research has a direct impact on the nation's income, whereas basic research has a direct impact only on the cost of carrying out applied research: applied research becomes "easier", cheaper, more likely to succeed, and so on, when the body of basic research available to society is bigger. Related to this is the difference in the "directness" of the link between research effort and the realisation of the beneficial effects of this effort. Simplifying somewhat, all research is uncertain, but, while in the case of applied research the uncertainty regards whether or not a certain line of research will be successful, that is whether or not a given, known problem is "solved", in the case of basic research it is also unknown in advance *where* a positive effect will emerge, if it does.[2] I capture this unpredictability with the assumption

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[1]See, for example, the definition used by the US National Science Foundation to classify expenditure: "basic research is defined as systematic study directed toward fuller knowledge or understanding of the fundamental aspects of phenomena and of observable facts without specific applications towards processes or products in mind." Conversely, "applied research is defined as systematic study to gain knowledge or understanding necessary to determine the means by which a recognized and specific need may be met." (NSB 2008, p 7).

[2]Nelson (1959 pp 301–2) gives several examples of basic research projects pursued as

of a completely diffuse link between pure and applied research: each applied research project is helped equally by the total amount of basic research undertaken in society. Basic research thus bestows an externality. This, however, does not create the appropriability problems which beset R&D activities carried out in profit maximising firms, well-understood by the literature since at least Arrow (1962). This is both because all effects of research are internal to the government, which funds research,[3] and because individuals and institutions doing research are not concerned with its monetary appropriability: their reward is the *production* of knowledge, not its financial exploitation, as has long been recognised (see Stephan 1996 for a comprehensive review).

The optimal funding structure for the centralised funding mechanism derived in Section 3 illustrates how information constraints force the government to use basic research as a reward to the more productive institutions to induce them to perform more applied research. This is inefficient, both because the marginal rate of return of funds is different across institutions, and because, for some institutions, the marginal social return is different for funds they allocate to basic and applied research. Moreover, institutions which could do research cheaply on a small scale are not funded at all.

The paper next shows how the optimal funding can be implemented in practice. I show that the dual funding system suggests itself naturally: all

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an end in themselves, which unexpectedly assists the solution of an apparently completely unrelated applied research problem. Among the more recent ones, Moody (1995) describes in detail the numerous strands of basic research which allowed the creation of the ubiquitous CD. A central plank of the theory of relativity, that light is bent by gravity, is also a building block of GPS navigation system (Haustein 2009). The abstract mathematical problem of covering a surface with symmetric tiles lies at the foundation of our understanding and exploitation of superconductors (Edelson 1992). Gauss's investigation into the distribution of prime number has led, with the contributions of many mathematical minds over the course of two centuries, to the possibility of unbreakable cryptographic codes, without which e-commerce would not be possible (du Satoy 2003). Table 3 in Gersbach et al (2009) has a longer and more systematic list. An empirical investigation of the link between basic research conducted in universities and commercial applications of the applied research it generated is in Jensen and Thursby (2001).

[3] Of course in an international context, some of the benefits determined by the expenditure of one country's taxpayers' money do accrue to individuals in different countries. This can be captured by an increase in the shadow cost of public funds relative to the value it would take in a closed economy.

institutions receive an identical "block grant", subject to a threshold level of applied research. More efficient institutions, those which can do this minimum amount of applied research spending less than the "block grant" can therefore use their savings to engage in basic research. When the social value of applied research is sufficiently high, additional funding is made available to institutions. To receive it, an institution must carry out additional applied research, and, crucially, the additional funding is lower than the cost of the additional applied research to be carried out: institutions need to "co-fund" any further applied research they wish to carry out. This is in contrast to the "cost-plus" approach favoured by funding agencies in the UK, and its intuitive explanation is that, since the government wants to push institutions to do applied research rather than basic research, even when the institution's cost of applied research is higher that the cost of doing basic research, it offers to fund the former, thus reducing an institution's cost.

The paper is organised as follows. Section 2 presents the model, and Section 3 the results. Section 4 shows how the policy can be implemented in practice, and Section 5 concludes. Mathematical proofs are in the Appendix.

2   The model

I model the publicly funded research sector of an economy. There is a continuum of institutions with the potential to do research their number normalised without loss of generality to 1. A government agency has the task of funding their activities. Institutions differ in their ability to spend public research funding productively. This ability is measured by a parameter $\theta \in [\underline{\theta}, \bar{\theta}] \subseteq \mathbb{R}$. The value of θ for each institution is exogenously given. The distribution of θ in the sector is described by a differentiable function $F(\theta)$, with density $f(\theta) = F'(\theta) > 0$, for $\theta \in (\underline{\theta}, \bar{\theta})$, and monotonic hazard rate, $\frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right) > 0$. The most natural interpretation for θ is the skill of an institution's scientists,[4]

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[4]In the simplest model, each institution is randomly assigned its staff. In reality, of course, institutions compete for staff, which would make θ endogenous. In a fully developed model of the academic labour market, one would need to take into account the fact that researchers prefer to join high quality institutions, and so competition among institutions might not be based exclusively on salaries. Palomino and Sákovics (2004) is a model which combines

but it can also encompass the institution's ability to supplement government funding with funds from private sources. These may include for example income from endowments, or, for universities, generated from students' tuition fees, even though I do not model explicitly any technological complementarity or financial cross-subsidisation between teaching and research (see for example De Fraja and Valbonesi 2008). Another example is of course the possible commercial exploitation of research. Two institutions which differ in this respect, for example because of their contacts with industry, of the effectiveness of their technology transfer office (or marketing department), will be characterised by different values of θ.

2.1 Basic and applied research.

If funded, an institution can carry out two kinds of research, basic and applied. I assume that applied research affects *directly* national income, whereas basic research affects it only indirectly via its effect on the productivity of applied research. Both applied and basic exert their influence via their aggregate amount, defined here as follows. Let $a(\theta)$ and $b(\theta)$ denote the average amount of applied and basic research, respectively, carried out in the institutions with productivity θ. I define by A and B the *total*[5] amount of applied research carried out in society. Therefore we have:

$$A = \int_{\underline{\theta}}^{\bar{\theta}} a(\theta) f(\theta) d\theta, \tag{1}$$

$$B = \int_{\underline{\theta}}^{\bar{\theta}} b(\theta) f(\theta) d\theta. \tag{2}$$

We can interpret A exactly as a standard Solow residual, and, taking other inputs as given, define national income as

$$Y(A), \tag{3}$$

with $Y'(A) > 0$, $Y''(A) \leqslant 0$. The link between a specific applied research project and the consequent increase in national income is left implicit. As an

competition among institutions (sports leagues in their paper), with externalities among its members (the individual teams).

[5]Notice that aggregate uncertainty disappears: even though specific research projects may be uncertain, then (1) and (2) denote the expected *and* actual amount of successful research.

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example, consider a project consisting in the development of a new therapy. It might be that the institution carrying out the project is a private profit-making pharmaceutical company receiving a government subsidy to research, or a research centre or a university selling a patent through a TTO.[6] In these cases a direct impact of research on national output can be established. Alternatively, if the line of applied research is not fully appropriable and the benefit is more diffuse, consumers or other firms might benefit directly, through new products or lower costs and prices. An example could be an improvement in communication technology, which benefit all users.

I follow Gersbach et al (2010), who posit that *aggregate* amount of basic research undertaken in society is a parameter of the function which gives the probability of a successful innovation in each of the continuum of industries where research is undertaken. Formally, in my model, a type θ research institution's cost of carrying out the amount a of applied research and the amount b of basic research is

$$\hat{c}\left(a, b, \theta, B\right). \tag{4}$$

I simplify the above with the assumption that applied and basic research enter the cost function in an additively separable manner. That is, $\hat{c}_{ab}\left(\cdot\right) = 0$ in the entire domain of $\hat{c}$. This reflects the unpredictability of the beneficial effects of basic research, which makes completely diffuse the externality from basic to applied research: each institution benefits equally from basic research carried out anywhere, and there is no complementarity between basic and applied research *within* an institution. I also assume that there are constant returns to scale in basic research, $\hat{c}_{bb}\left(\cdot\right) = 0$ in the domain of $\hat{c}$. The idea is that the cost of a basic research project, in relation to its probability to succed, is difficult to assess, making it hard for institutions to "rank" projects according to their "value for money". In view of these, the cost function simplifies to

$$\hat{c}\left(a, b, \theta, B\right) = c\left(a, \theta, B\right) + bc^{b}\left(\theta, B\right),$$

for some functions c and c^{b}.

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[6]The analysis of the role and effects of Technology Transfers Offices, outside the scope of this paper, can be found for example, in Macho-Stadler et al (2007) and in the references reported there.

Assumption 1 *For every $a, B \geqslant 0$, for every $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$, the functions $c(a, \theta, B)$ and $c^b(\theta, B)$ satisfy:*

1. $c_a(\cdot) > 0$, $c_\theta(\cdot) > 0$, $c_B(\cdot) < 0$, $c(0, \theta, B) = 0$.

2. $c_{aa}(\cdot) > 0$, $c_{BB}(\cdot) > 0$, $c_{a\theta}(\cdot) > 0$, $c_{Ba}(\cdot) \leqslant 0$.

3. $-c_B(a, \theta, 0) > 1$ *for every $a \geqslant 0$, for every $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$.*

4. $c^b(\theta, B) = 1$.

Assumption 1.1 simply defines θ as a measure of cost, captures the externality created by B, and rules out fixed costs. The second set of hypotheses are natural decreasing returns to scale assumptions ($c_{aa}(\cdot) > 0$ and $c_{BB}(\cdot) > 0$), and that a lower θ and more basic research decrease the marginal cost, as well as the total cost. Assumption 1.3 avoids unrewarding corner solutions by ensuring that if there is no basic research in society then a very small amount reduces the cost of research by more than it costs.

The last part of Assumption 1 states that the cost of doing basic research is independent of the productivity parameter, and then normalises it to 1, redefining if necessary the amount of basic research B. This assumption implies that all institutions are equally good at doing basic research, and, while it might be argued that this reflects the nature of basic research,[7] it does not seem to tally with stylised facts. In fact, I introduce it not for realism, but to separate incentive from efficiency considerations in the allocation of funding for basic research. I show below in Proposition 3 that, in conditions of imperfect information, low θ institutions, which are more efficient in carrying out *applied* research, do more *basic* research. This is not the case with symmetric information. In view of Assumption 1.4 it is clear that this is not because they are also better at basic research, but due to a different mechanism: the fact that the funding agency's information disadvantage forces it to offer funding that institutions can use to pay for basic research in order to induce those institutions that are efficient at applied research to do more of it than they would like.

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[7]Carried out by isolated group of researchers whose output is independent of their institutional affiliation, like archetypical Gregor Mendels experimenting on pea plants in an abbey in Brno.

2.2 Payoff functions

I assume that a research institution's objective is the maximisation of the total amount of research it carries out, r, subject to any constraints it must satisfy:

$$r(\theta) = a(\theta) + b(\theta). \qquad (5)$$

The additive form in (5) simply implies that the marginal rate of substitution between applied and basic research is constant. Any preference that institutions might have is normalised away, and the substantive part of the (5) is that institutional preferences between basic and applied research vary neither with their type θ, nor with the amount of basic and applied research they do.

The government chooses its research funding policy: the amount of funding to basic and applied research, and the way to distribute this funding across the various institutions. Formally, a research policy is a pair of functions, $\{t(\theta), a(\theta)\}_{\theta \in [\underline{\theta}, \bar{\theta}]}$, where $t(\theta)$ is the total funding given to institutions of type θ and $a(\theta)$ is the amount of applied research they need to do in order to receive that funding. Equivalently, since the amount of basic research in a type θ institution is simply the difference $t(\theta) - c(a(\theta), \theta, B)$, a policy can be written as $\{b(\theta), a(\theta)\}_{\theta \in [\underline{\theta}, \bar{\theta}]}$, the amount of basic and applied research in a type θ institution, or also as $\{r(\theta), a(\theta)\}_{\theta \in [\underline{\theta}, \bar{\theta}]}$, the amount of total and applied research in a type θ institution. The government's objective function is the total national income, reduced by the cost of funding the research sector, which includes a distortionary component, plus the non-monetary benefit of research (prestige, etc.). Formally, the government's payoff function is

$$k(A + B) + Y(A) - (1 + \lambda)T. \qquad (6)$$

where $k \geqslant 0$ is the weight of the non-monetary benefit of research, $\lambda > 0$ the shadow cost of public funds, and $T = \int_{\underline{\theta}}^{\bar{\theta}} t(\theta) f(\theta) d\theta$ the total funding to research. In view of (5), (2) can be replaced by:

$$B = \int_{\underline{\theta}}^{\bar{\theta}} [r(\theta) - a(\theta)] f(\theta) d\theta. \qquad (7)$$

The following assumption ensures that research is sufficiently important.

Assumption 2 *For every $A \geqslant 0$, $\frac{Y'(A) + k}{1 + \lambda} > 1$.*

That is, the marginal social benefit of *applied* research exceeds the marginal cost of *basic* research.

3 Results

3.1 Preliminaries

The viewpoint of this paper is normative: the government needs to choose how to distribute the research budget across institutions and to direct institutions' choice of the balance between applied and basic research. The government's maximisation problem is of course subject to information constraints, which are discussed in detail below. Before I present the results, it is convenient to define the amount of applied research which equates the marginal return on applied and basic research.

Definition 1 $a^* (\theta; B)$ *is value of a which solves*

$$c_a (a, \theta, B) = 1. \tag{8}$$

That is, $a^* (\theta; B)$ is the amount of applied research which maximises type θ institution's total research when the aggregate amount of basic research is B, provided the budget available to the institution is large enough not to constrain applied research; this can be defined as the *individually efficient expenditure on applied research*. Note that $\frac{\partial a^*(\cdot)}{\partial \theta} = -\frac{c_{a\theta}(\cdot)}{c_{aa}(\cdot)} < 0$ and $\frac{\partial a^*(\cdot)}{\partial B} = -\frac{c_{aB}(\cdot)}{c_{aa}(\cdot)} \geqslant 0$. That is, more efficient institutions have a higher individually efficient expenditure on applied research, and an increase in the level of basic research increases the individually efficient expenditure on applied research for all universities. This seems natural and has been dubbed "crowding in" of basic research (Malla and Gray 2005, p 434).

3.2 Perfect information

The first proposition gives the benchmark case in which the government fully and freely observe the productivity and the research activities of each institu-

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tion. Let $a_1(\theta)$, A_1 and B_1 be defined by:

$$c_a\left(a_1\left(\theta\right),\theta,B_1\right)=\frac{Y'\left(A_1\right)+k}{1+\lambda}, \tag{9}$$

$$A_1=\int_{\underline{\theta}}^{\bar{\theta}} a_1\left(\theta\right)f\left(\theta\right)d\theta, \tag{10}$$

$$\frac{k}{1+\lambda}=\int_{\underline{\theta}}^{\bar{\theta}} c_B\left(a_1\left(\theta\right),\theta,B_1\right)f\left(\theta\right)d\theta+1. \tag{11}$$

By Assumption 2, $\frac{Y'(A_1)+k}{1+\lambda}>1$, and so $a_1\left(\theta\right)>a^*\left(\theta;B\right)$.

Proposition 1 *If the government could observe perfectly the productivity of each institution and the amount of applied and basic research each institution carries out, it would choose: $a_1\left(\theta\right)$, and any function $r\left(\theta\right)\geqslant a_1\left(\theta\right)$ such that $\int_{\underline{\theta}}^{\bar{\theta}} r\left(\theta\right)f\left(\theta\right)d\theta=A_1+B_1$.*

The proofs of all results are relegated to the Appendix. Notice that, since $-c_B\left(a_1\left(\theta\right),\theta,0\right)>1>1-\frac{k}{1+\lambda}$, by virtue of Assumption 1.3, and $c_{BB}\left(\cdot\right)$ in Assumption 1.2, then B_1 determined in (11) is strictly positive.

Proposition 1 is straightforward: the government simply asks each institution to carry out a certain amount of applied research. Since $\frac{Y'(A_1)+k}{1+\lambda}>1$, by Assumption 2, this is more than $a^*\left(\theta;B\right)$, what each institution would choose if it were simply given a budget to spend as it pleases. This is what one would expect: applied research is more beneficial to the government than to institutions, and so the government want institutions to do more than they would like. By (9), the marginal cost of doing applied research is the same in every institution. This is efficient; if it were not the case, the government could transfer research from one institution to another and reduce the overall cost of a given total amount of applied research.

Notice that more efficient universities do more applied research: $a_1'\left(\theta\right)=-\frac{c_{a\theta}(\cdot)}{c_{aa}(\cdot)}<0$. They are better at it, so this is natural. Equally natural is the fact that the distribution of basic research across universities is a matter of indifference.[8] Since all institutions are equally productive at basic research,

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[8]Without Assumption 1.4, $a^*\left(\theta;B\right)$ would be defined not by (8), but by the solution in a to $c_a\left(a,\theta,B\right)=c^b\left(\theta,B\right)$; in this case the government would allocate basic research to the lowest cost institutions only, or, assuming also decreasing returns to scale in basic research,

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the government determines the total amount of basic research in (11) and then distributes it in any feasible way, that is in any way such that $b(\theta) \geqslant 0$ for every $\theta \in [\underline{\theta}, \bar{\theta}]$. Lastly, note that an increase in k and a reduction in λ increase $a_1(\theta)$ for every $\theta \in [\underline{\theta}, \bar{\theta}]$, thus increasing A_1 and B_1.

3.3   Information asymmetry.

I now consider a more realistic information structure. Specifically, I assume that the government can observe neither θ, an institution's productivity, nor b, the amount of basic research it does. Instead, the government can observe, and an external adjudicator can verify, whether an agreed level of applied research effort a has been performed.

This schematic assumption captures essential features of the two types of research effort discussed above. The funding agency, like any observer external to the institution, is able to verify whether or not resources destined to applied research have in fact been spent on applied research: evidence of expenditure on laboratories, data collection, research assistants' time and so on can be audited, even when, due to the uncertain nature of the research process, no tangible result is obtained. The same, however, is less true of basic research: if a funding body were to request an institution to destine a certain amount of resources to basic rather than applied research, against the institution's preferences, it would find it difficult to verify whether that request has been complied with: a requirement imposed on an institution to, say, hire a theoretical researcher can be easily circumvented. Similarly, requiring a researcher to do blue sky thinking must entail that she is given the freedom to choose any project that stimulates her curiosity: including, necessarily, applied projects, even though they are not specifically funded. In other words, it is easier to devote to applied research resources intended by the funding agency for basic research, than the other way round.

A related difficulty would emerge if a government tried to induce its preferred combination of applied and basic research by rewarding past achievement. It is much more difficult to do so for basic than for applied research.

in such a way to equalise the marginal cost of basic research. The other qualitative features of the analysis would not change.

Consider the latter. One can think that an institution carrying out a large portfolio of applied research projects, only some of which will succeed. The aggregate uncertainty in the portfolio cancels out, and the ex-ante expected output of the portfolio is approximately equal to the actual output, which, considering that applied research projects have a shorter time horizon, becomes observable relatively soon: if the government's funding agency rewards the institution for its output it is effectively rewarding effort and quality. The very nature of basic research is such that this is not true in its case. Some empirical evidence suggests that basic research expenditure is a more important productivity determinant than applied research (e.g., Mansfield (1980); Link (1981); Griliches 1986), and yet, the very long time gap and the often extremely tenuous link between research results and their impact (see the examples given in footnote 2) make it simply impossible to reward the observed success of basic research effort in a way that reflect its contribution to society. To sum up, research effort is more observable for applied research.[9] The formal implication for the model is that the government policy can impose a floor but not a ceiling on the amount of applied research: it can condition the amount of funding on observing at least a specified *minimum* amount of applied research; but the institution can "hide" its applied research if it does more than the specified minimum.[10]

The next result illustrates that, *per se*, this kind of unobservability does not limit the policy of the government.

Corollary 1 *If the government can only observe whether at least a certain amount of applied research has been carried out, but can observe perfectly the type of the institution, then it would choose $a_1(\theta)$ and any function $b(\theta) \geqslant 0$*

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[9]Note that peer review based formal evaluation mechanisms such as the Research Assessment Exercise in the UK are intended to assess the research effort of institutions, both in applied and in basic research. The current version of the exercise also attempts to measure the *impact* of research on society.

[10]As a specific example, consider the use of the budget of a social scientist (including her salary): suppose I want her to devote half of it to applied and half to basic research. I can ask her to spend some time collecting and organising, data, and knowing the technology she uses, I can determine that (at least) half her budget was indeed devoted to the task. But this is not so with the rest of her time: was she trying out different things with her data? or speculating on some abstract problems?

such that $\int_{\underline{\theta}}^{\bar{\theta}} b\left(\theta\right) f\left(\theta\right) d\theta = B_1.$

That is, the optimal policy when the government is unable to observe basic research is exactly as in Proposition 1, when it can observe the nature of the research carried out. The reason is straightforward: the government wants each institution to carry out the appropriate amount of applied research, and offers individualised contracts to each institution with this requirement. Since these contracts require a minimum amount of applied research higher than what the institution would do on its own $(a_1\left(\theta\right) > a^*\left(\theta; B\right))$, they are incentive compatible. Basic research, the cost of which is independent of the institution where it is carried out, is distributed in any feasible way.

This policy, however, cannot be implemented if the government cannot observe each institution's productivity parameter, θ. The reason is that Proposition 1 and Corollary 1 require each institution to choose a combination of basic and applied research effort such that applied research has a higher marginal cost than basic research, and so, if institutions were simply asked to do $a_1\left(\theta\right)$ and $b\left(\theta\right)$, they would have an incentive to claim to have a higher θ than they have. By doing so, they would be able to increase the total amount of research they do with their funding, as they can carry out less of the more costly applied research. Formally, presented with the link between a funding level and $a_1\left(\theta\right)$, a type θ institution would claim to be of type $\min\left\{a_1^{-1}\left(a^*\left(\theta; B\right)\right), \bar{\theta}\right\}$. In this way, its marginal cost of doing applied research is as near as possible to 1, its marginal cost of basic research.

3.4 Incentive Compatibility

In this subsection, I determine the constraints imposed by the information disadvantage of the government. I take the standard revelation approach. It is as if the government asked each institution to report its own type, and commits to imposing a vector of variables as functions of the reported type; by the revelation principle, the government cannot improve on the payoff it can obtain by restricting its choices to policies that satisfy the incentive compatibility constraint, that is the property that no institution has an incentive to misreport its type. This constraint is derived in Proposition 2. Recall that a policy can be written as $\left\{r\left(\theta\right), a\left(\theta\right)\right\}_{\theta \in \left[\underline{\theta}, \bar{\theta}\right]}$, with $r\left(\theta\right)$ and $a\left(\theta\right)$ the total and the applied

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research required of institution of type θ, $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$. Basic research follows from $r(\theta) = a(\theta) + b(\theta)$.

Proposition 2 *A policy* $\{r(\theta), a(\theta)\}_{\theta \in \left[\underline{\theta}, \overline{\theta}\right]}$ *is feasible and incentive compatible if it satisfies, for* $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$:

$$\dot{r}(\theta) = -c_\theta(a(\theta), \theta, B), \qquad r(\underline{\theta}) \text{ free}; r(\overline{\theta}) = 0, \qquad (12)$$

$$\dot{a}(\theta) \leqslant 0, \qquad (13)$$

$$a(\theta) - a^*(\theta; B) \geqslant 0, \qquad (14)$$

$$r(\theta) - a(\theta) \geqslant 0. \qquad (15)$$

3.5 The optimal funding policy

I am now in a position to present the government maximisation problem. This is the choice of a policy $\{r(\theta), a(\theta)\}$, which satisfies the constraints given in Proposition 2 and maximises the government objective function. As in Section 3.2, A and B, the aggregate amount of applied and basic research are best treated as parameters chosen by the government, subject of course to (1) and (7), their definition as the sum of applied and basic research carried out in all universities. Requirement (14), that $a(\theta) > a^*(\theta; B)$, implies that the government might want to exclude some institutions. As the proof shows, if it does so, it will exclude institutions with θ above a certain threshold. This threshold, $\theta_0 \in \left[\underline{\theta}, \overline{\theta}\right]$, is itself a choice variable.

The government's problem is therefore the following.

$$\max_{\substack{r(\theta), a(\theta), \\ A, B, \theta_0}} Y(A) + k(A + B) - (1 + \lambda) \int_{\underline{\theta}}^{\theta_0} \left[c(a(\theta), \theta, B) + r(\theta) - a(\theta)\right] f(\theta) d\theta$$

$$\text{s.t.: (12), (13), (14), (15), (7), (1).} \qquad (16)$$

I can now derive the optimal funding policy. This is based on three functions, $a^*(\theta; B)$ defined above, and $a^K(\theta; B, \beta)$ and $r^*(\theta; B, \theta_0)$, defined next. For given parameters $B > 0$, $\beta \geqslant 0$, let $a^K(\theta; B, \beta)$ be the solution in a of

$$c_a(a, \theta, B) = \frac{Y'(A) + k}{1 + \lambda} + \beta - \frac{\beta F(\theta)}{f(\theta)} c_{\theta a}(a, \theta, B). \qquad (17)$$

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The curve $a^K(\theta; B, \beta)$ is drawn in Figures 1 and 2, and is intuitively discussed following Corollary 2. Next, let $r^*(\theta; B, \theta_0)$ be the solution to the following differential equation:

$$\dot{r}(\theta) = -c_\theta(r(\theta), \theta, B), \qquad r(\theta_0) = a^*(\theta_0; B). \tag{18}$$

That is, for given B, $r^*(\theta; B, \theta_0)$ is the function $r(\theta)$ which satisfies the incentive compatibility constraint "shifted" to intersect $a^*(\theta; B)$ at $\theta = \theta_0$.

Assumption 3 (i) $c_\theta(\cdot) > \frac{c_{a\theta}(\cdot)}{c_{aa}(\cdot)}$; (ii) $c_\theta(\cdot) > \frac{c_{\theta\theta a}(\cdot)}{c_{\theta aa}(\cdot)} + \frac{c_{a\theta}(\cdot)}{c_{\theta aa}(\cdot)} \frac{\frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right)}{\frac{F(\theta)}{f(\theta)}}$, (iii) $c_{\theta aa}(\cdot) > 0$.

The first two statements in Assumption 3 impose restrictions on the cost structure. Loosely speaking, they require $c_\theta(\cdot)$ to be "large", that is that cost differences among institutions, measured by the parameter θ, are sufficient important. But θ is unobservable, and measures the differences in productivity among institutions, and therefore the first two statements in Assumption 3 simply require that information disadvantage of the government be sufficiently important. It is this disadvantage that makes the analysis relevant: in the extreme polar case where all research institutions are identical, the government's inability to observe their productivity is obviously not a problem. Of course, the considerable effort that funding agencies exert to ascertain the research potential of the research institutions they sustain suggests strongly that these differences are important in practice. The third statement is a regularity restriction.

Proposition 3 *Let Assumptions 1-3 hold. If problem (16) has a solution, then there exist $\tilde{\theta}, \theta_K \in (\underline{\theta}, \theta_0]$ with $\underline{\theta} < \theta_K \leqslant \tilde{\theta} \leqslant \theta_0$ such that:*

$$\text{if } \theta \in [\underline{\theta}, \theta_K) \text{ then } a(\theta) > a^*(\theta; B) \text{ and } b(\theta) > 0;$$
$$\text{if } \theta \in \left[\theta_K, \tilde{\theta}\right) \text{ then } a(\theta) = a^*(\theta; B) \text{ and } b(\theta) > 0;$$
$$\text{if } \theta \in \left[\tilde{\theta}, \theta_0\right] \text{ then } a(\theta) = a^*(\theta; B) \text{ and } b(\theta) = 0.$$

The following implication of Proposition 3 is worth illustrating formally, as it helps illustrating the optimal policy in a diagram.

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Corollary 2 *Let Assumptions 1-3 hold. If a solution to problem (16) exists, then there exists $A, B > 0$, $\theta_0 \in \left[\underline{\theta}, \overline{\theta}\right]$, and $\beta > 0$, such that:*

$$a\left(\theta\right) = \min\left\{\max\left\{a^*\left(\theta; B\right), a^K\left(\theta; B, \beta\right)\right\}, r^*\left(\theta; B, \theta_0\right)\right\}$$

$$b\left(\theta\right) = \max\left\{r^*\left(\theta; B, \theta_0\right) - \max\left\{a^*\left(\theta; B\right), a^K\left(\theta; B, \beta\right)\right\}, 0\right\}$$

for $\theta \in [\underline{\theta}, \theta_0]$, and $a\left(\theta\right) = b\left(\theta\right) = 0$ for $\theta \in \left(\theta_0, \overline{\theta}\right]$.

The optimal policy described in Proposition 3 and Corollary 2 is depicted in Figure 1. In each panel of the Figure, the vertical axis measures the amount of research at the optimal policy, and the horizontal axis an institution's productivity parameter, θ. Only institutions with θ below θ_0, the intersection of $r^*\left(\theta; B, \theta_0\right)$ and $a^*\left(\theta; B\right)$, receive any government funding. In each panel, there are three different curves. The solid thin line is the locus $r^*\left(\theta; B, \theta_0\right)$: by Proposition 2, it represents the total amount of research carried out by a type θ institution. The dotted line is locus $a^*\left(\theta; B\right)$, and the dashed line is $a^K\left(\theta; B, \beta\right)$. By Corollary 2, the amount of *applied* research each institution does is given by the higher of these two curves, if it is below $r^*\left(\theta; B, \theta_0\right)$, and otherwise by $r^*\left(\theta; B, \theta_0\right)$ itself. In the latter case, the institution does no basic research. In both panels, the red thick curve is the amount of applied research carried out by a type θ institution, and the distance between the latter curve and $r^*\left(\theta; B, \theta_0\right)$, shaded in grey in the diagrams, is the amount of basic research carried out by a type θ institution.

The panels of Figure 1 differ in the position of the curve $a^K\left(\theta; B, \beta\right)$. This determines three possible patterns of complementary slackness of the constraints in Problem 16, indicated by the white numbers in a black disk. In region 1, both (14) and (15) are slack; in region 2, constraint (14) is binding. In region 3, (15) is binding. The conceptual difference between regions 1 and 2 is that research institutions in region 2 choose their "preferred" combination of applied and basic research, and those in region 1 are required/incentivised to do more than this amount. In region 3 there are research institutions which only do applied research.

When Assumption 2 is violated, applied research is less valuable; this pushes down the curve $a^K\left(\theta; B, \beta\right)$, and, if a solution exists, it will still satisfy Proposition 3 and Corollary 2. However, the relative position of the three curves is
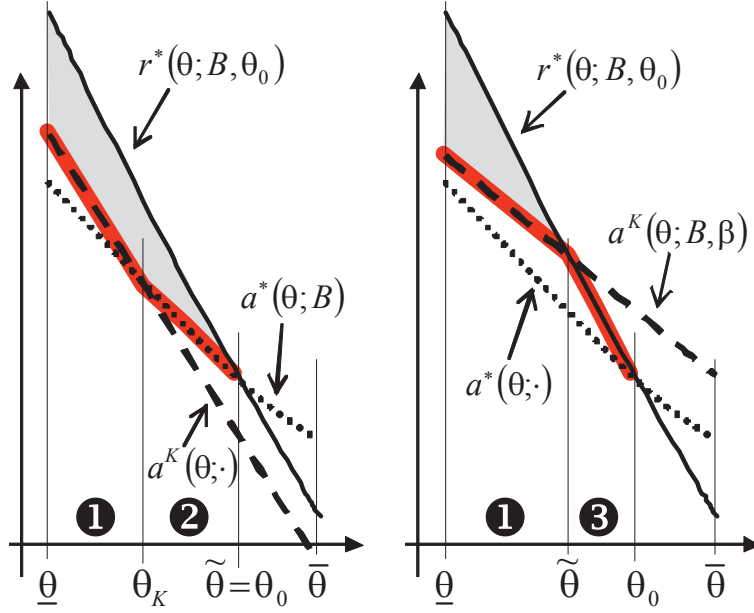
Figure 1: Applied and basic research. High social value of applied research.

now illustrated by Figure 2: the most productive research institutions now do carry out their preferred level of applied research, and if there are institutions which are given an incentieve to do more than this, as in the right hand side panel, they are the medium θ institutions.

These Figures illustrate how the pattern of funding is determined by the position of the curve $a^K (\theta; B, \beta)$. From (17), it is evident that this is affected by four factors: the direct effect of applied research on national income, $Y'(A)$; the direct "prestige" effect of research on the policy maker's payoff, k; the shadow cost of public funds λ; and finally, the endogenously determined overall effect of basic research on institutions' cost of doing applied research, β. The first three simply shift the dashed curve up and down in a parallel fashion (leaving aside the indirect effect through B). Thus, other things equal, increases in k and in $Y'(A)$ and decreases in λ all increase the amount of applied research, and decrease the amount of basic research. The sign of the effect for k and λ follows from the fact that the cost of applied research is higher that that of basic research, therefore, if research becomes more desirable, higher k, or cheaper to fund, lower λ, more of the "expensive" type, applied research will be done.

The term β determines the distortionary effect of information asymmetry. It is 0 with perfect information, as can be seen in the comparison between (17) and (9). Notice that it changes the amount of basic research done by the most efficient institution: superficially, this might appear in contrast to the "efficiency at the top" principle. In fact, information asymmetry affects the total amount of basic research that it is optimal to carry out, and therefore the socially optimal amount of applied research that the most efficient institution should do. A positive value of β increases the amount done by more efficient institutions, and reduces (more) the amount carried out by less efficient ones. For higher values of θ, however this effect is reduced by the incentive effect: in order to induce institutions to self-select, it is necessary to prevent high type universities from pretending to be low type, and thus doing less applied research and devote funds to the cheaper basic research: reducing sufficiently the amount of applied research makes this less attractive for a low θ institution. Basic research is used to provide incentives for institutions to carry out applied research in excess of $a^* (\theta; B)$, via a link between total funding and the amount of applied research done. This also explains why the number of institutions funded is reduced by asymmetric information. High cost institutions, which would do some research if the government could observe their type, cannot however be funded when the government has imperfect information, lest more efficient institutions "pretend" to be inefficient to avoid doing research in excess of their efficient level $a^* (\cdot)$, as the government would like them to, and increase their spending on basic research.

The next Section describes in detail how the link between applied research and total funding can be implemented in practice. Before, I consider a special case, which illustrates the role of the basic research externality.

Assumption 4 *The cost function* $c(a, \theta, B)$ *is additively separable in* (a, θ) *and* B: *there exist* $\hat{c}(a, \theta)$ *and* $\zeta(B)$ *such that* $c(a, \theta, B) = \hat{c}(a, \theta) + \zeta(B)$ *for every* (a, θ, B).

In words, B affects only the fixed cost of doing applied research.

Corollary 3 *If assumption 4 holds, then* $c_a \left(a^K (\cdot), \underline{\theta}, B \right) = 1 + \left(\frac{Y'(A)}{1+\lambda} + \zeta'(B) F(\theta_0) \right)$.
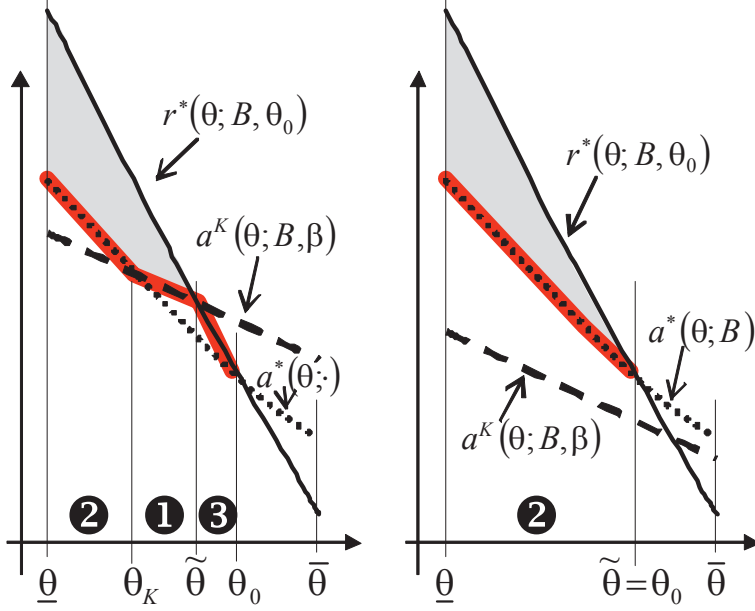
Figure 2: Applied and basic research. Low social value of applied research.

In contrast, recall that for the curve $a^*(\theta; B)$, it is $c_a(a^*(\cdot), \underline{\theta}, B) = 1$. Therefore by how much applied research is pushed above the efficient level in the most productive institutions, that is by how much the initial point of the dashed curve $a^K(\cdot)$ exceeds that of the the curve $a^*(\theta; B)$ depends on the extent by which, $\frac{Y'(A)}{1+\lambda}$, the marginal benefit of an increase in applied research, exceeds $\zeta'(B) F(\theta_0)$, the marginal cost of an increase in basic research.

4 Implementation

This section investigates how a central funding agency can implement in practice the optimal policy derived in Proposition 2. As I have assumed, this agency is constrained by the fact that basic research is unobservable: if an institution wants, it is able to divert to applied research funding intended for basic research. All an agency can therefore do is to provide a link between a target amount of applied research carried out and the total amount of funding an institution receives.

Formally, I want to derive the function $C(a)$, which gives the total funding

received as a function of the total amount of applied research carried out in an institution. Since there is a one-to-one relationship between θ and a, this is a well defined function.

The shape of this function depends on which the three regions identified in Proposition 2 the optimal choice belongs. To see this, consider an institution of type θ which, given policy $\{r(\theta), a(\theta)\}_{\theta \in [\underline{\theta}, \bar{\theta}]}$ chooses $r(\theta)$, $a(\theta)$ and therefore it obtains total funding $t(\theta)$. In region 1, the total funding received by this type θ institution is

$$t(\theta) = c\left(a^K(\theta; B, \beta), \theta, B\right) + \left[r^*(\theta; B, \theta_0) - a^K(\theta; B, \beta)\right].$$

The first term is the cost of carrying out $a^K(\theta; B, \beta)$ amount of applied research and the terms in the square brackets the cost of basic research. For fixed B and β, let $\theta^K(a; B, \beta)$ be the inverse of the function $a^K(\theta; B, \beta)$: $\theta^K(a; B, \beta)$ is the value of θ such that $a^K(\theta; B, \beta) = a$. Next, consider an institution which, faced with a schedule $C(a)$ chooses to carry out a (minimum) amount a of applied research (which, if the policy is incentive compatible, has therefore type $\theta^K(\theta; B, \beta)$). The total funding it receives is given by:[11]

$$C(a) = c\left(a, \theta^K(a; B, \beta), B\right) + r^*\left(\theta^K(a; B, \beta); B, \theta_0\right) - a. \qquad (19)$$

Corollary 4 *If $a(\theta) = \theta^K(a; B, \beta)$, then $C(a)$ is increasing. $C(a)$ is convex if and only if $\frac{\partial a^*(\theta; B)}{\partial \theta} > \frac{\partial a^K(\theta; B, \beta)}{\partial \theta}$.*

Therefore $C(a)$ is convex if the relative slope of the dashed and dotted curves is as in the LHS of Figure 1, concave if it is as in the RHS.

The same procedure can be applied to derive the shape of $C(a)$ in the other regions. In region 2, let $\theta^*(a; B)$ be the inverse function of $a^*(\theta; B)$, so that total funding is given by:

$$C(a) = c\left(a, \theta^*(a, B), B\right) + r^*\left(\theta^*(a, B); B, \theta_0\right) - a. \qquad (20)$$

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[11] Notice that, faced with (19), a type θ institution does indeed want to carry out precisely $a = a^K(\theta; B, \beta)$ applied research. To see this, note that it will solve

$$\max_{a \geq 0} \max \left\{a + [C(a) - c(a, \theta, B)], a^*(\theta; B)\right\},$$

where $C(a)$ is given by (19). The first order condition for the above is $c_a\left(a, \theta^K(a; B, \beta), B\right) = c_a(a, \theta, B)$.
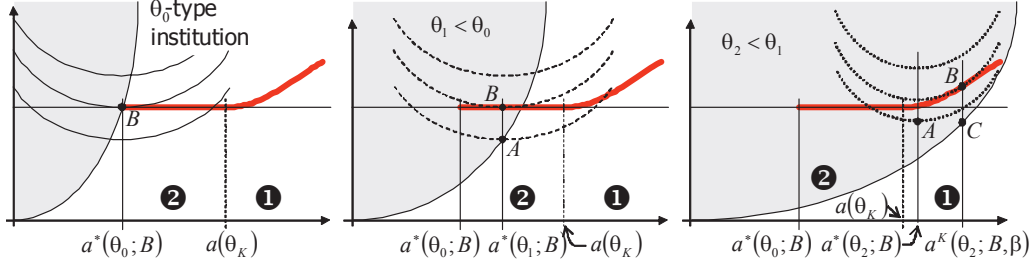
Figure 3: Implementation: The LHS of Figure 1.

Corollary 5 *If $a(\theta) = a^*(\theta; B)$, then $C(a)$ is constant.*

Finally region 3. Here with the same argument used for region 1, I show that the function is again increasing and convex.

Corollary 6 *If $a(\theta) = r^*(\theta; B, \theta_0)$, then $C(a)$ is increasing and convex. Moreover, at the boundary between region 1 and 3 the slope of $C(a)$ is increasing in a.*

Having determined the slope of the function $C(a)$, consider Figure 3, to see how the funding agency can implement it in practice. The Figure shows the cartesian space, with the amount of applied research on the horizontal axis, and the total funding on the vertical axis. The function $C(a)$, defined for $a \in \left[a^*(\theta_0; B), \max\left\{a^K(\underline{\theta}; B, \beta), a^*(\underline{\theta}; B)\right\}\right]$ is the red thick line: points on this locus represent combinations of funding and applied research which the funding agency allows research institutions to choose from. It is drawn for the case where the relevant region for the high type research institutions is of type 2 (that is for the LHS panel of Figure 1). Here, as shown in Corollary 5, funding as a function of total applied research is constant. Also drawn on the diagram are the indifference curves of for three institution types, and their "feasible set", the combinations of funding and the amount of applied research which an institution can carry out with that funding. Consider first a type θ_0 institution, shown on the LHS panel. Its indifference curves are the solid thin lines (they all reach a minimum at $a = a^*(\theta_0; B)$: totally differentiate $a + t - c(a, \theta_0, B)$ to see this), and its feasible set is the grey shaded area (this is the set $\{(a, t) \in \mathbb{R}^+ | c(a, \theta_0, B) \leqslant t\}$). There is only one point available for

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this institution, the point on the red locus and on the "feasible set", namely $(a^* (\theta_0; B), C (a^* (\theta_0; B)))$. Not so however for more productive research institutions: take type $\theta_1 \in (\theta_K, \theta_0)$, illustrated in the middle panel. Its indifference curves are the dashed lines, and its feasible set again the grey area, clearly bigger than a type θ_0 institution's. It can choose any point in the grey area and on the red line. The best among such points is $(a^* (\theta_1; B), C (a^* (\theta_1; B)))$, point B in the diagram. Notice that, the required level of applied research, $a^* (\theta_1; B)$, will cost it only $c (a^* (\theta_0; B), \theta_0, B)$, the vertical height of point A, which is less that $C (a^* (\theta_1; B))$: after paying for applied research, it is "left" with some funding which it will spend on basic research. This has marginal cost of 1, rather than applied research, which, if pushed above $a^* (\theta_1; B)$, would have a marginal cost exceeding 1. A type θ_1 institution therefore carries out an amount of basic research measured by the vertical distance between points A and B.

Finally consider a very efficient institution, one with cost parameter $\theta_2 < \theta_K$. Its efficient level of applied research is $a^* (\theta_2; B)$, as shown by point A in the RHS panel of the Figure. This is the level it would choose if funding were constant. But the optimal policy is designed so that this institution does more than this amount, as the funding agency offers increasing funding for research institutions which exceed their efficient level of applied research: faced with the red schedule, a type θ_2 research institution chooses the combination that allows it to be on the highest possible indifference curve, namely tangency point B in the diagram.[12] Note that the total cost incurred by this institution to carry out the amount of applied research $a^K (\theta_2; B, \beta)$, say point C, is less than its total amount of funding. It will spend the difference to pay for basic research, which therefore is given by the distance between B and C: the vertical height of point C measures the amount spent on applied research, and so the distance

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[12]When the curve $C (a)$ is convex, as in Figure 3, then the tangency point is a local, and hence a global, maximum. To see this, note that, at the tangency point $(a_2, C (a_2))$, with $a_2 = a^K (\theta_2; B, \beta)$, the slope of the indifference curve is given by $c_a (a, \theta, B) - 1$. The slope of the funding schedule is given by (A12). In a neighbourhood of a_2, we have:

$$c_a (a_2 + \varepsilon, \theta_2, B) - c_a \left(a_2 + \varepsilon, \theta^K (a_2 + \varepsilon; B, \beta), B\right) = c_{a\theta} (a_2 + \varepsilon, \theta_3, B) \left(\theta_2 - \theta^K (a_2 + \varepsilon; B, \beta)\right).$$

For $\varepsilon > 0$ (resp $\varepsilon < 0$), the above is positive (negative), as $a^K (\cdot)$ is decreasing and so $\theta^K (\cdot)$ is too. Clearly if the curve $C (a)$ is concave, then the tangency point is a maximum.
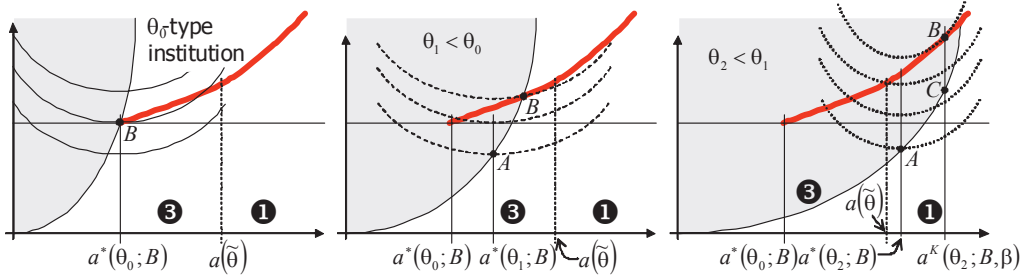
Figure 4: Implementation: The RHS of Figure 1.

between B and C is the amount of basic research that a type θ_2 does.

When the relative position of the curves $a^*\left(\theta_0; B\right)$ and $a^K\left(\theta; B, \beta\right)$ is instead that shown in the RHS panel of Figure 1 the optimal funding can be implemented by the schedule illustrated in Figure 4. This differ from the top panel only in that the initial part of the schedule is also increasing. Notice how a type θ_1 institution, illustrated in the middle panel of the Figure, spends all of its budget on applied research, and still does more than its efficient level. As before, a very low cost institution, $\theta_2 < \theta_K$, which would choose to spend an amount given by the height of point A on applied research if it were on a fixed budget, chooses instead point B, and spends an amount equal to the vertical height of point C on applied research and the distance between C and B on basic research.

To end this discussion, it is worth noting that when the social value of applied research is low, so that the position of the curve $a^K\left(\theta; B, \beta\right)$ is as depicted in the RHS panel of Figure 2, then the optimal policy is implemented simply by a policy of constant funding: all research institutions that agree to carry out at least $a^*\left(\theta_0; B\right)$ applied research, receive the funds necessary to pay for it, which they can use in any way they choose. In this case the diagram of the funding schedule looks exactly the same as the initial portion of the red line on Figure 3, from $a^*\left(\theta_0; B\right)$ to $a\left(\tilde{\theta}\right)$.

I consider next which mechanism can in practical terms be used to induce institutions to choose one point on the red line. This is a dual funding mechanism: there is a fixed part, and a output related part. Specifically suppose that all institutions which can carry out at least $a^*\left(\theta_0; B\right)$ applied research receive

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a lump sum $t_0 = c\left(a^*\left(\theta_0; B\right), \theta_0, B\right)$. In addition, institutions can apply to have specific projects funded through a grant. However, not all institutions can apply for these grants, but, in order to qualify to apply institutions need to carry out at least $a^*\left(\theta_K; B\right)$ applied research. In this case it would receive grant funding governed by the formula

$$g\left(a\right) = C\left(a + a^*\left(\theta_K; B\right)\right) - c\left(a^*\left(\theta_0; B\right), \theta_0, B\right)$$

where $g\left(a\right)$ is the amount of grant awarded for agreeing to carry out a units of applied research, in addition to the qualifying level $a^*\left(\theta_K; B\right)$. It is worth noting that, at least for amounts in excess of the qualifying level below a given level, the amount awarded does not cover the additional cost. Formally.

Corollary 7 *Suppose $\theta_K < \tilde{\theta} = \theta_0$. There exists $\Delta > 0$ such that there exist $\theta_\Delta > \theta_K$ such that $g\left(\Delta\right) < c\left(a^*\left(\theta_K; B\right) + \Delta, B, \theta\right) - c\left(a^*\left(\theta_K; B\right), B, \theta\right)$ for every $\theta \in \left(\theta_K, \theta_\Delta\right)$.*

Graphically, consider Figure 3. According to the Corollary, the slope of the red line in a neighbourhood of $a^*\left(\theta_K; B\right)$, which gives the additional funding received by an institution that just exceeds the qualifying level $a^*\left(\theta_K; B\right)$, is less than the slope of the frontier at the same point, which given the additional cost incurred by such an institution to exceed by a small amount the qualifying level of applied research, $a^*\left(\theta_K; B\right)$.

The mechanism illustrated in the above paragraph constitutes an instance of the principle of co-funding by the grant funding agency and the institutions. This is in contrast with the idea of full economic costing, used, among others, by the research councils in the UK (RCUK/UUK 2010). According to this principle, grants are over funded, that is the amount of the grant awarded for applied research exceeds the cost to carry out that research. The rationale is precisely to ensure that there is no cross-subsidisation among an institution's various activities. My results here shows that the optimal policy is more subtle and that the benefit of avoiding cross-subsidisation must be balanced with the benefit derived from designing incentives to delegate funding decisions to the institutions with the private information necessary to take allocation decisions efficiently. The principle of co-funding may be reversed for higher values of applied research, that is for very efficient institutions, and indeed, unit funding

might become impossible, if the curve $C(a)$ is concave. For the other regions it is possible to derive similar results, but the taxonomy of each case is of limited interest.

5 Concluding Remarks

Developed countries spend around one fifth of their R&D expenditure on basic research (Gersbach 2009, p 114). Should they spend more? Less? The UK government funds research via two separates channels, quality related funding and research grants from the research councils, in a proportion of roughly 2/3 and 1/3. Is this ratio "right"? Also, research grants are less evenly distributed: the top 25 universities received 85% of the research grant funding, and 75% of quality related funding. Are these proportions "right"? Government agencies typically award research grants on a cost plus principle, whereas charitable bodies favour co-funding of research activities. Which is better?

The theoretical guidance necessary to answer these questions, and more generally to establish a microeconomic foundation to any empirical study of research funding is relatively scant. In this paper, I offer a framework for the provision of this guidance. I develop a model built on the ideas that research institutions can devote their research effort to basic or applied research, which are in a hierarchical relationship, that there are differences in research productivity among institutions, and that the government aims to distribute its funding in the socially preferred manner, which in general differs from the preferences of individual institutions.

References

Aghion, P., Dewatripont, M., Stein, J. C., 2008. Academic freedom, private-sector focus, and the process of innovation. RAND Journal of Economics 39, 617–635.

Arrow, K., 1962. The economic implications of learning by doing. Review of Economic Studies 29, 155–173.

De Fraja, G., Valbonesi, P., 2008. The design of the university system. CEPR Discussion Paper 7038, London.

du Sautoy, M., 2003. The Music of the Primes. HarperCollins, London.

Edelson, E., 1992. Physics: Quasicrystals and superconductors: Advances in condensed matter physics. In: Greenwood, A. (Ed.), Science at the Frontier. National Academy Press, Washington DC, pp. 233–254.

Evenson, R. E., Kislev, Y., 1976. A stochastic model of applied research. Journal of Political Economy 84, 765–782.

Gersbach, H., 2009. Basic research and growth policy. In: Foray, D. (Ed.), The New Economics of Technology Policy. Edward Elgar, Cheltenham, UK, pp. 113–121.

Gersbach, H., Schneider, M., Schneller, O., 2010. Optimal mix of applied and basic research, distance to frontier, and openness. CEPR Discussion Paper 7795, London.

Gersbach, H., Sorger, G., Amon, C., 2009. Hierarchical growth: Basic and applied research. ETH Economics Working Paper Series 09/118, Zurich.

Griliches, Z., 1986. Productivity, R&D, and basic research at the firm level in the 1970's. American Economic Review 76, 141–154.

Haustein, M., 2009. Effects of the theory of relativity in the GPS. Chemnitz University of Technology.

Jensen, R., Thursby, M., 2001. Proofs and prototypes for sale: The licensing of university inventions. American Economic Review 91, 240–259.

Laffont, J.-J., Tirole, J., 1993. A Theory of Incentives in Procurement and Regulation. MIT Press, Cambridge, Massachusetts.

Leonard, D., van Long, N., 1992. Optimal Control Theory and Static Optimization in Economics. Cambridge University Press, Cambridge, UK.

Link, A. N., 1981. Basic research and productivity increase in manufacturing. American Economic Review 71, 1111–1112.

Macho-Stadler, I., Pérez-Castrillo, D., Veugelers, R., 2007. Licensing of university inventions: The role of a technology transfer office. International Journal of Industrial Organization 25, 483–510.

Mansfield, E., 1980. Basic research and productivity increase in manufacturing. American Economic Review 70, 863–873.

Moody, R. V., 1995. Why curiosity driven research? http://www.math.mun.ca/∼edgar/moody.html.

National Science Board, 2008. Research and development: Essential foundation for U.S. competitiveness in a global economy. National Science Foundation.

Nelson, R. R., 1959. The simple economics of basic scientific research. Journal of Political Economy 67, 297–306.

OECD, 2009. OECD science, technology and industry scoreboard 2009. OECD, Paris.

Palomino, F., Sákovics, J., 2004. Inter-league competition for talent vs. competitive balance. International Journal of Industrial Organization 22, 783–797.

RCUK/UUK Task Group, 2010. Financial sustainability and efficiency in full economic costing of research in UK higher education institutions. London.

Stephan, P. E., 1996. The economics of science. Journal of Economic Literature 34, 1199–1235.

Strandburg, K. J., 2005. Curiosity-driven research and university technology transfer. The Berkeley Electronic Press.

Appendix

Proof of Proposition 1. Divide the government objective function (6) by $(1 + \lambda)$, substitute (7) and the value of T to write the optimization problem as:

$$\max_{a(\theta), r(\theta), A, B} \frac{Y(A) + k(A+B)}{1+\lambda} - \int_{\underline{\theta}}^{\bar{\theta}} [c(a(\theta), \theta, B) + r(\theta) - a(\theta)] f(\theta) \, d\theta, \quad \text{(A1)}$$

$$\text{s.t. (1) and (7).}$$

Next, following Leonard and van Long (1992, p 190), write (7) and (1) as

$$\dot{b}^0(\theta) = [r(\theta) - a(\theta)] f(\theta), \qquad b^0(\underline{\theta}) = 0, \qquad b^0(\bar{\theta}) = B, \qquad \text{(A2)}$$

$$\dot{a}^0(\theta) = a(\theta) f(\theta), \qquad a^0(\underline{\theta}) = 0, \qquad a^0(\bar{\theta}) = A. \qquad \text{(A3)}$$

Ignoring for the moment the constraint $r(\theta) - a(\theta) \geqslant 0$, the Lagrangian for (A1) is:

$$\mathcal{L}(\cdot) = -[c(a(\theta), \theta, B) + r(\theta) - a(\theta)] f(\theta) + [\sigma a(\theta) + (1 - \beta)(r(\theta) - a(\theta))] f(\theta),$$

where σ and $(1 - \beta)$ are the Lagrange multipliers for constraints (A3) and (A2). I write the multiplier of (A2) as $(1 - \beta)$ to lighten notation. The first order conditions give (see Leonard and van Long, 1992, Theorem 7.11.1, p 255):

$$\frac{\partial \mathcal{L}}{\partial a(\theta)} = [-c_a(a(\theta), \theta, B) + 1 + \sigma - (1 - \beta)] f(\theta) = 0,$$

$$\frac{\partial \mathcal{L}}{\partial r(\theta)} = (-1 + (1 - \beta)) f(\theta) = 0,$$

$$\sigma = \frac{k + Y'(A)}{1 + \lambda},$$

$$1 - \beta = \frac{k}{1 + \lambda} - \int_{\underline{\theta}}^{\bar{\theta}} c_B(a(\theta), \theta, B) f(\theta) \, d\theta.$$

Rearranging, $\beta = 0$, and the result follows. ∎

Proof of Corollory 1. The problem in this case is the same as (A1), with the added constraint $a(\theta) \geqslant a^*(\theta; B)$. At the solution of problem (A1) derived in Proposition 1, this constraint is slack and so the solution found there remains a solution for the new problem. ∎

Proof of Proposition 2. Notice first of all that $b(\theta)$ must be non-negative, and so (15) must hold. For policy $\{r(\theta), a(\theta)\}$ to be incentive compatible, every type θ institution must (weakly) prefer to report to be of type θ that to pretend of to be of

type x, for every $x \in [\underline{\theta}, \overline{\theta}]$. This determines (12). To see how, begin to note that by choosing to report type $x \in [\underline{\theta}, \overline{\theta}]$, institution of type θ receives an amount of funds for applied research $a(x)$ and a total research funding $c(a(x), x, B) + r(x) - a(x) = c(a(x), x, B) + b(x)$. Total cost is observable, and so the institution needs to choose research levels such that its total cost equals the last expression. So if a institution of type θ has reported type x, it will choose research levels a_L and b_L (L stands for "lying") which maximise

$$(a_L, b_L) = \arg\max_{a,b} a + b,$$

$$\text{s.t.: } c(a, \theta, B) + b = c(a(x), x, B) + r(x) - a(x),$$

$$a \geqslant a(x).$$

Or

$$a_L = \max_a a + c(a(x), x, B) + r(x) - a(x) - c(a, \theta, B),$$

$$\text{s.t.: } a \geqslant a(x).$$

Which has solution $a_L = a^*(\theta; B)$ if $a^*(\theta; B) \geqslant a(x)$, and $a_L = a(x)$ if $a^*(\theta; B) < a(x)$. That is:

$$a_L = \max \{a^*(\theta; B), a(x)\},$$

$$b_L = c(a(x), x, B) + r(x) - a(x) - c(a_L, \theta, B),$$

and yields payoff:

$$\varphi(x, \theta) = \begin{cases} a^*(\theta; B) + c(a(x), B, x) + r(x) - a(x) - c(a^*(\theta; B), \theta, B), \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{if } a^*(\theta; B) \geqslant a(x), \\ a(x) + c(a(x), x, B) + r(x) - a(x) - c(a(x), \theta, B), \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{if } a^*(\theta; B) < a(x); \end{cases}$$

consider local maxima:

$$\frac{\partial \varphi(x, \theta)}{\partial x} = \begin{cases} (c_a(a(x), x, B) - 1)\dot{a}(x) + c_\theta(a(x), x, B) + \dot{r}(x), \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{if } a^*(\theta; B) \geqslant a(x), \\ c_a(a(x), x, B)\dot{a}(x) + c_\theta(a(x), x, B) + \dot{r}(x) - c_a(a(x), \theta, B), \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{if } a^*(\theta; B) < a(x). \end{cases}$$

For incentive compatibility, this needs to be maximised at $x = \theta$. Evaluating the above at $x = \theta$, we get:

$$\frac{\partial \varphi(x, \theta)}{\partial x} = \begin{cases} c_\theta(a(x), x, B) + \dot{r}(x) & \text{if} \quad a^*(\theta; B) \geqslant a(x), \\ c_\theta(a(x), x, B) + \dot{r}(x) & \text{if} \quad a^*(\theta; B) < a(x). \end{cases}$$

A2

The first line holds because the optimal a when $a^*(\theta; B) \geqslant a(x)$ is $a^*(\theta; B)$ and $c_a(a^*(\theta; B), x, B) - 1 = 0$: this establishes (12).

Now (13): following Laffont and Tirole (1993, p 121), for a policy to be incentive compatible it must be that

$$\frac{\partial^2 \varphi(\theta, x)}{\partial \theta \partial x} \geqslant 0.$$

We have

$$\frac{\partial^2 \varphi(\theta, x)}{\partial \theta \partial x} = -c_{a\theta}(a(x), \theta, B) \dot{a}(x) \geqslant 0,$$

given our assumption that $c_{a\theta}(a(x), \theta, B) > 0$, (13) must hold.

Finally, (14). This follows from

$$\frac{d(c(a(\theta), \theta, B) + r(\theta) - a(\theta))}{d\theta} \leqslant 0. \tag{A4}$$

This is the constraint that total funding be decreasing in θ. If it were not the case, then an institution could simply claim to have a higher θ than it has, thus receiving a higher funding, which it could spend on unobservable basic research. Expand (A4):

$$c_a(a(\theta), \theta, B) \dot{a}(\theta) + c_\theta(a(\theta), \theta, B) + \dot{r}(\theta) - \dot{a}(\theta) \leqslant 0$$

which, using (12), becomes

$$[c_a(a(\theta), \theta, B) - 1] \dot{a}(\theta) \leqslant 0,$$

since $\dot{a}(\theta) \leqslant 0$, $c_a(a(\theta), \theta, B)$ must exceed 1, which is (14). ∎

Proof of Proposition 3. Begin by noting that $a(\theta) \in [a^*(\theta; B), r(\theta)]$, and therefore a solution exists only for values of θ such that $r(\theta) \geqslant a^*(\theta; B)$, that is for $\theta \leqslant \theta_0$ at the candidate solution. This is because, by virtue of Assumption 3(i), $r(\theta) > a^*(\theta; B)$ to the left of their intersection, θ_0. As in Proposition 1, divide the maximand of problem (16) by $(1 + \lambda)$, and construct the Lagrangian.

$$\begin{aligned}
\mathcal{L}(\cdot) = &- [c(a(\theta), \theta, B) + r(\theta) - a(\theta)] f(\theta) - \mu(\theta) c_\theta(a(\theta), \theta, B) + \\
&\gamma(\theta)(a(\theta) - a^*(\theta; B)) + \pi(\theta)(r(\theta) - a(\theta)) + \\
&[(1 - \beta)(r(\theta) - a(\theta)) + \sigma a(\theta)] f(\theta),
\end{aligned} \tag{A5}$$

where $\mu(\theta)$, $\gamma(\theta)$, $\pi(\theta)$, are the multipliers associated to constraints (12), (14), and (15) respectively. As before $(1 - \beta) > 0$ and $\sigma > 0$ are the multipliers for (A2) and

(A3). I have ignored constraint (13): it will be seen to be satisfied at the solution found when it is ignored. The first order conditions on $r(\theta)$ and $a(\theta)$ are given by:

$$-\frac{\partial \mathcal{L}}{\partial r(\theta)} = \dot{\mu}(\theta) = \beta f(\theta) - \pi(\theta), \qquad \mu(\underline{\theta}) = 0, \qquad \mu(\bar{\theta}) \text{ free;} \qquad (A6)$$

$$\frac{\partial \mathcal{L}}{\partial a(\theta)} = [-c_a(a(\theta), \theta, B) + \beta + \sigma] f(\theta) + \gamma(\theta) - \pi(\theta)$$
$$- \mu(\theta) c_{\theta a}(a(\theta), \theta, B) = 0. \qquad (A7)$$

(A6) has solution:

$$\mu(\theta) = \beta F(\theta) - \Pi(\theta), \qquad (A8)$$

having defined $\Pi(\theta) = \int_{\underline{\theta}}^{\theta} \pi(\tilde{\theta}) d\tilde{\theta}$. The first order conditions for A and B are the same as in Proposition 1, giving $\sigma = \frac{k+Y'(A)}{1+\lambda}$. Expand the condition on $(1-\beta)$, using (A8), and the definition of $a^*(\theta; B)$, which implies $\frac{\partial a^*}{\partial B} = -\frac{c_{aB}(\cdot)}{c_{aa}(\cdot)}$:

$$1 - \beta = \frac{k}{1+\lambda} + \int_{\underline{\theta}}^{\theta_0} \left[-c_B(\cdot) f(\theta) - (\beta F(\theta) - \Pi(\theta)) c_{\theta B}(\cdot) + \gamma(\theta) \frac{c_{aB}(\cdot)}{c_{aa}(\cdot)} \right] d\theta.$$

Integration by parts gives:

$$1 - \beta = \frac{k}{1+\lambda} - c_B(a(\theta_0), \theta_0, B) F(\theta_0) + (1-\beta) \int_{\underline{\theta}}^{\theta_0} F(\theta) c_{\theta B}(\cdot) d\theta$$
$$+ \int_{\underline{\theta}}^{\theta_0} \left[\Pi(\theta) c_{\theta B}(\cdot) + \gamma(\theta) \frac{c_{aB}(\cdot)}{c_{aa}(\cdot)} \right] d\theta,$$

and so

$$1 - \beta = \frac{\frac{k}{1+\lambda} - c_B(a(\theta_0), \theta_0, B) F(\theta_0)}{1 - \int_{\underline{\theta}}^{\theta_0} F(\theta) c_{\theta B}(\cdot) d\theta} + \frac{\int_{\underline{\theta}}^{\theta_0} \left[\Pi(\theta) c_{\theta B}(\cdot) + \gamma(\theta) \frac{c_{aB}(\cdot)}{c_{aa}(\cdot)} \right] d\theta}{1 - \int_{\underline{\theta}}^{\theta_0} F(\theta) c_{\theta B}(\cdot) d\theta}. \qquad (A9)$$

From (A7) we obtain the optimality condition for $a(\theta)$.

$$c_a(a(\theta), \theta, B) = \frac{Y'(A) + k}{1+\lambda} + \beta + \frac{\gamma(\theta) - \pi(\theta)}{f(\theta)} - \frac{\beta F(\theta) - \Pi(\theta)}{f(\theta)} c_{\theta a}(a(\theta), \theta, B). \qquad (A10)$$

Next notice that $\beta \geqslant 0$. To see this, notice that $(1-\beta)$ measures the benefit of relaxing the constraint $b(\theta) \geqslant 0$, which has a cost, measured in social value of monetary units, of 1. Notice that the funding agency can always increase $b(\theta)$ if it wants, because it can simply increase the funding to all research institutions, and, since at the optimum they all do at least $a^*(\theta; B)$, they all prefer to spend the additional funding on basic research. Therefore the benefit of increasing $b(\theta)$ cannot exceed the cost at the optimum: $(1-\beta) \leqslant 1$.

A4

Now define the function $a_\Pi^K(\theta; B, \beta)$ as the solution in a of

$$c_a(a, \theta, B) = \frac{Y'(A) + k}{1 + \lambda} + \beta - \frac{\beta F(\theta) - \Pi}{f(\theta)} c_{\theta a}(a, \theta, B). \tag{A11}$$

If $\Pi = 0$, then $a_\Pi^K(\theta; B, \beta) = a^K(\theta; B, \beta)$ and if $\Pi > 0$, then $a_\Pi^K(\theta; B, \beta) > a^K(\theta; B, \beta)$, since $c_{\theta a}(\cdot) > 0$.

Next notice that depending on the combination of complementary slackness for constraints (14) and (15), a value of $a(\theta)$ belongs to one of four possible regions:

1. $a(\theta) - a^*(\theta; B) > 0$ and $r(\theta) - a(\theta) > 0$. Therefore, $\gamma(\theta) = \pi(\theta) = 0$, which means $r(\theta) > a(\theta) > a^*(\theta; B)$, and in this region, $r(\theta) = r^*(\theta; B, \theta_0)$, $a(\theta) = a_{\Pi(\theta)}^K(\theta; B, \beta)$.

2. $\gamma(\theta) > 0$ and $r(\theta) - a(\theta) > 0$. Here, $a(\theta) - a^*(\theta; B) = 0$ and $\pi(\theta) = 0$, and so $r(\theta) = r^*(\theta; B, \theta_0)$, $a(\theta) = a^*(\theta; B)$.

3. $a(\theta) - a^*(\theta; B) > 0$ and $\pi(\theta) > 0$. In this region $\gamma(\theta) = 0$ and $r(\theta) = a(\theta) = r^*(\theta; B, \theta_0)$.

4. $\gamma(\theta) > 0$ and $\pi(\theta) > 0$. Here, $r(\theta) = r^*(\theta; B, \theta_0) = a^*(\theta; B) = a(\theta)$, and therefore this region is just the single intersection point between $a^*(\theta; B)$ and $r^*(\theta; B, \theta_0)$.

As a preliminary step, I show that

$$\text{if } \theta \in \left[\underline{\theta}, \tilde{\theta}\right) \text{ then } a(\theta) > 0 \text{ and } b(\theta) > 0;$$
$$\text{if } \theta \in \left[\tilde{\theta}, \theta_0\right] \text{ then } a(\theta) > 0 \text{ and } b(\theta) = 0.$$

Proposition 3 requires that $\underline{\theta}$ belongs to regions 1 or 2, that is that $a(\underline{\theta}) \in [a^*(\underline{\theta}, B), r^*(\underline{\theta}; B, \theta_0))$. Suppose by contradiction that $a(\underline{\theta}) = r^*(\underline{\theta}; B, \theta_0)$. Then $b(\underline{\theta}) = 0$ in $\left[\underline{\theta}, \tilde{\theta}\right]$ for some $\tilde{\theta} > \underline{\theta}$. Notice next that it cannot be $\tilde{\theta} = \theta_0$, otherwise $b(\theta) = 0$ in $\left[\underline{\theta}, \bar{\theta}\right]$ and so $B = 0$, against the Inada condition. That is, there is $\tilde{\theta} > \theta_0$ such that $a(\theta) = a_{\Pi(\theta)}^K(\theta; B, \beta) < r^*(\theta; B, \theta_0)$ in a right neighbourhood of $\tilde{\theta}$, with of course $a\left(\tilde{\theta}\right) = r^*\left(\tilde{\theta}; B, \theta_0\right) = a_{\Pi(\theta)}^K\left(\tilde{\theta}; B, \beta\right)$. Now we show that at any intersection between $r^*(\theta; B, \theta_0)$ and $a_{\Pi(\theta)}^K(\theta; B, \beta)$ the latter is *less steep* than $r^*(\theta; B, \theta_0)$, and thus we obtain a contradiction: if $a_{\Pi(\theta)}^K(\theta; B, \beta)$ is less steep than $r^*(\theta; B, \theta_0)$ then it must be *above* it in a right neighbourhood of $\tilde{\theta}$.

Lemma A1 $a_\Pi^K(\theta; B, \beta) > r^*(\theta; B, \theta_0)$ *for* $\theta > \tilde{\theta}$.

Proof. To see this, compare $a_{\Pi}^{K}(\theta, B, g)$ and $r^{*}(\theta; B, \theta_0)$ at their intersection. Since we are assuming that $a(\theta) > r^{*}(\theta; B, \theta_0)$, we have $\pi(\theta) = \frac{d\Pi}{d\theta} = 0$. Moreover, since $a(\theta)$ is above $a^{*}(\theta; B)$ in $\left[\underline{\theta}, \tilde{\theta}\right]$, it must be $\beta F(\theta) - \Pi(\theta) > 0$ in $\left[\underline{\theta}, \tilde{\theta}\right]$. Next totally differentiate (A11):

$$\left[c_{aa}(\cdot) + \frac{gF(\theta) - \Pi}{f(\theta)}c_{\theta aa}(\cdot)\right]da+$$
$$\left[c_{a\theta}(\cdot) + \frac{gF(\theta) - \Pi}{f(\theta)}c_{\theta\theta a}(\cdot) + c_{a\theta}(\cdot)g\frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right)\right]d\theta = 0.$$

Hence:

$$\frac{\partial a_K(\theta; B, g)}{\partial \theta} = -\frac{c_{a\theta}(\cdot) + \frac{gF(\theta)-\Pi}{f(\theta)}c_{\theta\theta a}(\cdot) + c_{a\theta}(\cdot)g\frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right)}{c_{aa}(\cdot) + \frac{gF(\theta)-\Pi}{(1+\lambda)f(\theta)}c_{\theta aa}(\cdot)}.$$

I need to verify that the following holds:

$$-\frac{c_{a\theta}(\cdot) + \frac{gF(\theta)-\Pi}{f(\theta)}c_{\theta\theta a}(\cdot) + c_{a\theta}(\cdot)g\frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right)}{c_{aa}(\cdot) + \frac{gF(\theta)-\Pi}{f(\theta)}c_{\theta aa}(\cdot)} > -c_{\theta}(\cdot).$$

By Assumption 3, $c_{\theta aa}(\cdot) > 0$, and so I can multiply through and rearrange:

$$\frac{\frac{gF(\theta)-\Pi}{f(\theta)}}{c_{aa}(\cdot)}\left(c_{\theta\theta a}(\cdot) + c_{a\theta}(\cdot)\frac{\frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right)}{\frac{F(\theta)}{f(\theta)}} - c_{\theta aa}(\cdot)c_{\theta}(\cdot)\right) < c_{\theta}(\cdot) - \frac{c_{a\theta}(\cdot)}{c_{aa}(\cdot)}.$$

Again, by Assumption 3, the RHS is positive and the LHS is negative. Therefore, at their intersection, $\frac{\partial a_K(\cdot)}{\partial \theta} > \frac{\partial r^{*}(\cdot)}{\partial \theta}$, that is $r^{*}(\cdot)$ is steeper, and so it is below $a^{K}(\cdot)$ in a right neighbourhood of their intersection. The contradiction establishes the Lemma. ∎

The Proposition now follows immediately. ∎

Proof of Corollary 2. Proposition 3 shows that $a(\theta)$ is one of $a^{*}(\theta; B)$, $a^{K}(\theta; B, \beta)$ or $r^{*}(\theta; B, \theta_0)$. Moreover, since it must lie between $a^{*}(\theta; B)$ and $r^{*}(\theta; B, \theta_0)$, it can only equal $a^{K}(\theta; B, \beta)$ – intersections excepted – between them. The second line follows from the first. ∎

Proof of Corollary 3. Omitted. ∎

Proof of Corollary 4. Differentiate (19) with respect to a, using (12):

$$C'(a) = c_a\left(a, \theta^{K}(a; B, \beta), B\right) - 1. \tag{A12}$$

The above is positive because $a^K(\theta; B, \beta)$ exceeds $a^*(\theta; B)$. C is therefore increasing. For the second part of the statement, expand $C''(a)$:

$$C''(a) = c_{aa}(\cdot) + c_{a\theta}(\cdot) \frac{\partial \theta^K(a; B, \beta)}{\partial a}.$$

This is positive if $-\frac{c_{a\theta}(\cdot)}{c_{aa}(\cdot)} = \frac{\partial a^*(\theta; B)}{\partial \theta} > \frac{\partial a^K(\theta; B, \beta)}{\partial \theta}$. $\blacksquare$

Proof of Corollary 5. The derivative of (20) is:

$$C'(a) = c_a(\cdot) + c_\theta(\cdot) \frac{\partial \theta^*(a; B)}{\partial a} + \frac{\partial r^*(\theta^*(a; B); B, \theta_0)}{\partial \theta} \frac{\partial \theta^*(a; B)}{\partial a} - 1 = 0,$$

as $c_a(\cdot) = 1$ along $a^*(\theta; B)$. $\blacksquare$

Proof of Corollary 6. Let $\theta^r(a; B, \theta_0)$ be the inverse function of $r^*(\theta; B, \theta_0)$, and total funding is given by (recall that $b(\theta) = 0$ in this region):

$$C(a) = c(a, \theta^r(a, B, \theta_0), B). \tag{A13}$$

Differentiation with respect to a yields:

$$C'(a) = c_a(\cdot) + \frac{c_\theta(\cdot)}{\frac{\partial r^*(\cdot)}{\partial \theta}} = c_a(\cdot) - 1.$$

Since $r^*(\theta; B, \theta_0) > a^*(\theta; B)$ except at θ_0, the above is positve in $\left(\tilde{\theta}, \theta_0\right)$. To establish convexity, take $C''(a)$:

$$C''(a) = c_{aa}(\cdot) + c_{a\theta}(\cdot) \frac{\partial \theta^r(a; B, \theta_0)}{\partial a},$$

which is positive as $-\frac{c_{a\theta}(\cdot)}{c_{aa}(\cdot)} > \frac{\partial r^*(\theta; B, \theta_0)}{\partial \theta} = -c_\theta(\cdot)$.

For the second part of the statement, note that, in region 3 (that is to the left of their intersection), the slope of $C(a)$ is $c_a(a, \theta^r(a; B, \theta_0), B) - 1$. In region 1, namely to the right of their intersection, the slope is $c_a(a, \theta^K(a; B, \beta), B) - 1$. Consider a right neighbourhood of their intersection: the difference in slope is

$$c_a(a, \theta^r(a; B, \theta_0), B) - c_a(a, \theta^K(a; B, \beta), B)$$
$$= c_{a\theta}(a, \theta_3, B)(\theta^r(a; B, \theta_0) - \theta^K(a; B, \beta)). \tag{A14}$$

This is positive, since $\theta^r(a; B, \theta_0) - \theta^K(a; B, \beta) > 0$, establishing the statement. $\blacksquare$

Proof of Corollary 7. Omitted. $\blacksquare$

A7